

Section 8.3: Properties of Logarithms

The properties of logarithms (blue box-page 412) should be memorized. They are very useful for solving exponential and logarithmic equations. Please avoid the following common mistake when using the quotient property.

$$\text{True: } \log_a u - \log_a v = \log_a \frac{u}{v} \qquad \text{However: } \log_a u - \log_a v \neq \frac{\log_a u}{\log_a v}$$

Following, the derivations of the logarithmic properties are provided. Spend some time studying them. Notice that these properties occur because of the relationship between logarithms and exponents.

$$\begin{aligned} \text{Proof Set Up:} \quad & \text{Let } x = \log_a u \Rightarrow a^x = u \\ & \text{and } y = \log_a v \Rightarrow a^y = v \end{aligned}$$

$$\begin{aligned} \text{I)} \quad \log_a uv &= \log_a a^x a^y && \text{(by substitution)} \\ &= \log_a a^{x+y} && \text{(property of exponents)} \\ &= x + y && \text{(by substitution)} \\ &= \log_a u + \log_a v && \text{(by substitution)} \end{aligned}$$

$$\begin{aligned} \text{II)} \quad \log_a \frac{u}{v} &= \log_a \frac{a^x}{a^y} && \text{(by substitution)} \\ &= \log_a a^{x-y} && \text{(property of exponents)} \\ &= x - y && \text{(by substitution)} \\ &= \log_a u - \log_a v && \text{(by substitution)} \end{aligned}$$

$$\begin{aligned} \text{III)} \quad \log_a u^n &= \log_a (a^x)^n && \text{(by substitution)} \\ &= \log_a a^{nx} && \text{(property of exponents)} \\ &= nx && \text{(by substitution)} \\ &= n \log_a u && \text{(by substitution)} \end{aligned}$$

Study Examples 1-5 which demonstrate how to use the properties to expand and to condense logarithmic expressions.

Assignment 8.3: #5,7,9,15,17,23,25,31,37,39,41,43,45